

Quantum transport in randomly diluted quantum percolation clusters in two dimensions

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We study the hopping transport of a quantum particle through finite, randomly diluted percolation clusters in two dimensions. We investigate how the transmission coefficient T behaves as a function of the energy E of the particle, the occupation concentration p of the disordered cluster, the size of the underlying lattice, and the type of connection chosen between the cluster and the input and output leads. We investigate both the point-to-point contacts and the busbar type of connection. For highly diluted clusters we find the behavior of the transmission to be independent of the type of connection. As the amount of dilution is decreased we find sharp variations in transmission. These variations are the remnants of the resonances at the ordered, zero-dilution, limit. For particles with energies within $0.25 \leq E \leq 1.75$ (relative to the hopping integral) and with underlying square lattices of size 20×20 , the configurations begin transmitting near $p_\alpha = 0.60$ with T against p curves following a common pattern as the amount of dilution is decreased. Near $p_\beta = 0.90$ this pattern is broken and the transmission begins to vary with the energy. In the asymptotic limit of very large clusters we find the systems to be totally reflecting except when the amount of dilution is very low and when the particle has energy close to a resonance value at the ordered limit or when the particle has energy at the middle of the band.

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I. INTRODUCTION

In classical systems a particle can traverse around a configuration of barriers as long as there is an energetically available path. However, in systems where quantum mechanical effects cannot be ignored due to the wavelike nature of the particle, constructive and destructive interference can occur whenever there are paths of different lengths available to the particle. The transport of a particle is therefore significantly influenced by quantum interference in systems where quantum mechanical effects cannot be ignored. In a previous paper [1] we focused on the effects of quantum interference in the hopping transport of a particle through finite, ordered square lattices. In this paper we extend our studies to include the transport of a particle through finite disordered clusters in two dimensions (2D) where the disorder is introduced by random dilution.

The question of whether disordered clusters in 2D are always insulating or not is controversial. One-parameter scaling theory [2] states that there is no metallic state in non-interacting 2D disordered systems. All states should be localized in an infinitely large and disordered 2D system at zero temperature. In weak localization, states are logarithmically localized, while in strong localization, states are exponentially localized. In the presence of disorder, states are either weakly or strongly localized depending on the amount of disorder. However, results from recent experiments on dilute low-disordered Si MOSFET and GaAs/AlGaAs heterostructures show hints of a metallic behavior and a metal-to-insulator tran-

sition in 2D disordered systems [3, 4, 5]. Several subsequent experiments on different 2D disordered systems also produce results hinting at a metal-to-insulator transition. For a review of these experiments see Abrahams *et al.* [6] Furthermore, experiments done on self-assembled quantum dots on Ga[Al]As heterostructures [7] also suggest a metal-to-insulator transition. Since one-parameter scaling theory deals only with non-interacting particles, it is possible that a metallic behavior can occur as a result of the interplay between inter-particle interactions and the disorder. There is also the question of whether the type of disorder chosen is relevant. One of the classic and intensively investigated model of hopping transport of non-interacting particles in disordered systems is the Anderson model of localization [8]. The particle in this model is governed by the tight-binding Hamiltonian

$$H_A = \sum_i \epsilon_i |i\rangle \langle i| + \sum_{\langle ij \rangle} v_{ij} (|i\rangle \langle j| + |j\rangle \langle i|), \quad (1)$$

where the $|i\rangle$ and $|j\rangle$ are tight-binding basis functions centered on sites i and j , respectively, $v_{ij} = 1$ if i and j are nearest-neighbor sites and $v_{ij} = 0$ otherwise, and the on-site energies ϵ_i are randomly chosen in some range $|\epsilon_i| \leq W$. A particle traverses the lattice by hopping from a site to a nearest neighbor site. As the range W is increased a transition from conducting to localized states occurs for systems of dimension $d \geq 3$. However, all states are localized for systems of dimension two or below. The type of disorder in this model therefore can not give rise to the observed metallic behavior in 2D disordered systems.

The disorder in Eq. (1) lies in the on-site energies ϵ_i while the underlying lattice is ordered. A variant of the Anderson model is the quantum percolation model [9, 10] wherein the on-site energies ϵ_i are held constant while the underlying lattice is a disordered cluster from percolation theory [11]. Since the underlying cluster is disordered not every nearest neighbor site is available for the particle to hop onto. In matrix representation the disorder in the Anderson model is located along the diagonal of the Hamiltonian while in quantum percolation the disorder is manifest at off-diagonal locations.

There is a long-standing question whether the Anderson model and the quantum percolation model yield the same type of localization behavior and thus, belong to the same universality class. For disordered clusters in three dimensions it is widely agreed that there is a transition from extended to localized states in quantum percolation [12, 13, 14, 15, 16]. For disordered clusters in 2D, however, there is no clear consensus whether such a transition exists in quantum percolation. Several groups of researchers including those using the dlog Padé approximation method [17], real space renormalization method [18], or studying the inverse participation ratio [19] found a transition from exponentially localized states to non-exponentially localized states for the site concentration that ranges within $0.73 \leq p_q \leq 0.87$. There are, however, studies that do not find evidence of such a transition. Studies by numerically calculating the conductance and making use of the one-parameter scaling hypothesis [20], by using the vibration-diffusion analogy [21], by finite-size scaling analysis and by transfer matrix methods [22], and vector recursion technique [23] found no evidence of a transition. A study by Inui *et al.* [24] found all states to be localized except for those with particle energies at the middle of the band and when the underlying lattice is bipartite, such as a square lattice. It is therefore not clear whether the Anderson model and the quantum percolation model produce the same type of localization behavior and if not, whether quantum percolation may be relevant to the experimentally observed conducting behavior in disordered systems in 2D.

There are three adjustable parameters in quantum percolation: the energy E of the incident particle, the concentration p for the random dilution of the lattice, and the size of the system. In this paper we present our studies and results by varying the values of all three parameters.

II. COUPLING THE LEADS TO THE CLUSTER AND DETERMINING THE CONDUCTANCE

In a previous paper [1] we studied the effects of quantum interference in the transmission of a particle through ordered, i.e., undiluted, square lattices. To study the transport properties we connected semi-infinite chains to the square lattices. The incident particle was then set to propagate from one chain, pass through the square lat-

tice, and then transmit through to the other chain. We used the quantum percolation model and set the constant on-site energy $\epsilon_i = 0$ in Eq. (1). Transmission and reflection coefficients were then determined from the eigenstates of the whole system including the chains. From the transmission and reflection coefficients the conductance may be calculated by making use of the Landauer-Büttiker formalism [25]. For transport through ordered square lattices we found transmission and reflection resonances whenever the energy of the incident particle was close to a doubly-degenerate eigenvalue of the square lattice. We also found the type of connection chosen between the chains and the square lattice to strongly influence the transport characteristics. Here we follow the same method described above in determining the transport properties of disordered clusters.

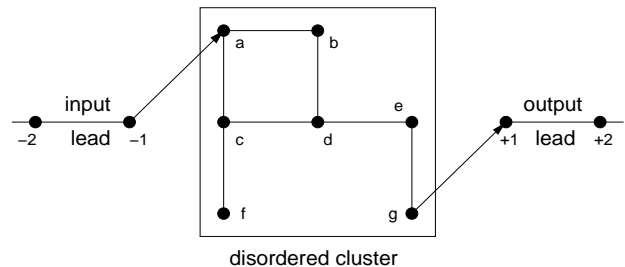


FIG. 1: An illustration of a disordered cluster on a 3×3 lattice attached to the input and output leads through point-to-point contacts. Beside each site is its unique label.

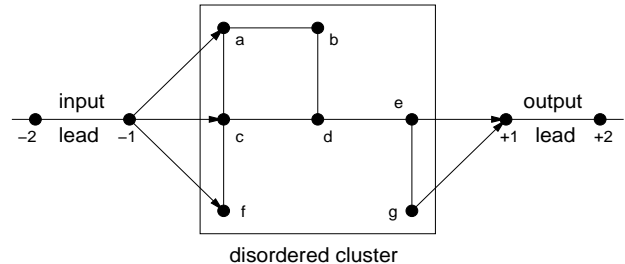


FIG. 2: An illustration of a disordered cluster on a 3×3 lattice attached to the input and output leads through a busbar connection. Beside each site is its unique label. Notice that in contrast to the point-to-point contacts there are multiple connections between the cluster and the leads in a busbar.

In this paper the disordered clusters are constructed using site percolation on a square lattice [11]. There are several ways of attaching the semi-infinite chains to the disordered clusters. Shown in Figs. 1 and 2 are two possible ways of attaching the chains. We label the chain where the particle is incident from as the input lead and the chain where the particle transmit through as the output lead. A point-to-point contacts type of connection is shown in Fig. 1. The input lead is attached to the top-leftmost site while the output lead is attached to the bottom-rightmost site in the disordered cluster. The particle is incident through the input lead, passes through

the cluster, and then transmits through the output lead. If the cluster is not spanning it is not possible to complete the connection from the input to the output lead. The incident particle is then hindered from propagating through the cluster. The minimum necessary requirement therefore for the particle to reach the output lead is for the cluster to be spanning. In site percolation on a square lattice the critical concentration of occupied sites is $p_c = 0.5927$. In quantum percolation, if there is indeed a transition from localized to extended states as the concentration of sites is increased then this transition should occur above the classical percolation threshold p_c .

A busbar type of connection between the cluster and the leads is shown in Fig. 2. The input lead is attached to all the sites at the left side of the cluster while the output lead is attached to all the sites at the right side of the cluster. In contrast to the point-to-point contacts, there are multiple connections between the leads and the cluster in a busbar. A physical realization of a busbar can be a set-up wherein the contacts connecting the disordered cluster to the current source are large enough to encompass and attach to the whole side of the cluster.

There are other ways of connecting the disordered cluster to the input and output leads. We have, however, chosen the point-to-point contacts and the busbar because at the ordered limit, when the cluster becomes a fully occupied lattice, these two types of connections are complementary in the sense that point-to-point contacts maintain the bipartite symmetry of the square lattice while the busbar, because of its multiple connections, breaks that symmetry [1].

To determine the transport properties of the disordered clusters we follow the method described by Daboul *et*

al. [17] Setting the constant on-site energy $\epsilon_i = 0$, the quantum percolation Hamiltonian becomes

$$H_{qp} = \sum_{\langle ij \rangle} v_{ij} (|i\rangle \langle j| + |j\rangle \langle i|). \quad (2)$$

We then attach the semi-infinite chains to the cluster. The transmission and reflection coefficients can then be determined from the resulting eigenvalue equation $H_{qp}\psi = E\psi$. However, this is an infinitely-sized problem and to reduce it to a finite one Daboul *et al.* made the following ansatz:

$$\begin{aligned} \psi_{-(n+1)} &= e^{-inq} + r e^{inq}, \\ \psi_{+(n+1)} &= t e^{inq}, \end{aligned} \quad (3)$$

where $n = 0, 1, 2, \dots$. Notice that shown beside each site in Figs. 1 and 2 is its unique label. Sites belonging to the cluster are labeled alphabetically. Sites belonging to the input chain are labeled by negative integers while those belonging to the output chain are labeled by positive integers. In the ansatz above $\psi_{-(n+1)}$ is the part of the wavefunction along the input chain and $\psi_{+(n+1)}$ is the part of the wavefunction along the output chain. The t and r are the transmission and reflection amplitudes, respectively. What the ansatz implies, therefore, is that the incoming plane wave is partially reflected back through the input chain and partially transmitted out to the output chain.

Using the ansatz in Eq. (3) we can reduce the infinitely-sized eigenvalue problem into a finite one. For example, for the busbar configuration shown in Fig. 2 the eigenvalue problem in matrix representation reduces to

$$\begin{pmatrix} -E + e^{iq} & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & -E & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -E & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & -E & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & -E & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -E & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & -E & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -E & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & -E + e^{iq} \end{pmatrix} \begin{pmatrix} 1 + r \\ \psi_a \\ \psi_b \\ \psi_c \\ \psi_d \\ \psi_e \\ \psi_f \\ \psi_g \\ t \end{pmatrix} = \begin{pmatrix} e^{iq} - e^{-iq} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}. \quad (4)$$

The connection between the cluster and the leads is manifest in the perimeter of the 9×9 square matrix above. The inner 7×7 inner sub-matrix involves the connections among sites in the cluster and is therefore independent of the type of connection between the cluster and the leads. In order for the ansatz to lead to valid solutions, the following relation between q and E must be satisfied along the chains:

$$e^{-iq} + e^{iq} = E. \quad (5)$$

The reflection coefficient $R = |r|^2$ and transmission coefficient $T = |t|^2$ can then be calculated from Eqs. (4) and (5) and by choosing a value for the incident particle's energy E . Note that because of the constraint condition, i.e., Eq. (5), along the chains the particle's energy is now restricted to be within $-2 \leq E \leq 2$. Also, although the problem now involves a finite matrix, effectively it is still an infinite system. Therefore, the energy E is continuous and any value within the range $[-2, 2]$ leads to a

valid solution.

Eq. (4) is in the form of a linear equation $Ax = b$ with x as the unknown. We numerically determine x exactly by solving the inverse, A^{-1} , using singular value decomposition [26] and then multiplying it to b . For a given lattice size, $L \times L$, as we increase the occupation concentration p the size of the disordered cluster, on average, increases as well. Consequently, the size of the linear problem to be solved increases. For a disordered cluster of size N the size of the matrix A to be solved is $(N + 2)^2$. In this paper the largest clusters we investigate are those embedded in lattices of size 30×30 . We repeat the calculations of the transmission amplitude t and reflection amplitude r for 1000 realizations of the disorder and then take the average of $|t|^2$ and $|r|^2$ to get the average transmission coefficient T and reflection coefficient R , respectively.

III. NUMERICAL RESULTS

We investigate the characteristics of the transmission coefficient by varying the energy E of the incident particle, the site occupation concentration p of the disordered clusters, and the size $L \times L$ of the lattice. Shown in Fig. 3 is a plot of the transmission coefficient T as a function of the energy E of the incident particle. The clusters are connected to the input and output leads through point-to-point contacts. In every plot shown in this paper each data point is an average over 1000 realizations of disorder configurations.

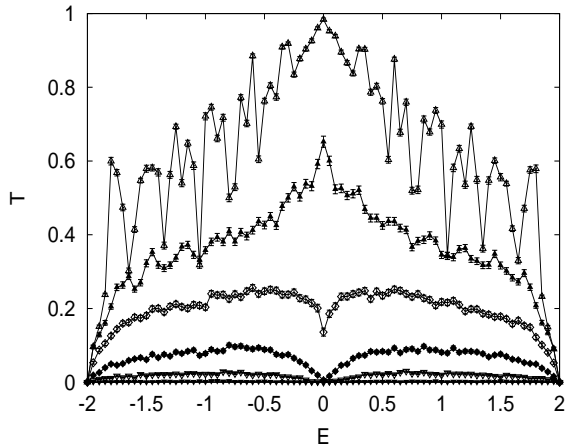


FIG. 3: Plot of the transmission coefficient T as a function of the incident particle's energy E for disordered clusters on 20×20 lattices with point-to-point contacts between the leads and the cluster. The disorder concentrations are $p = 0.60$ (\blacktriangledown), $p = 0.70$ (\triangledown), $p = 0.80$ (\blacklozenge), $p = 0.90$ (\diamond), $p = 0.95$ (\blacktriangle), and $p = 0.99$ (\triangle).

In Fig. 3 notice that for $p = 0.60, 0.70$, and 0.80 when the clusters are highly disordered the transmission curves are shaped like an inverted w with a dip at the middle of the band. The curves then change shape as the oc-

cupation concentration goes from $p = 0.90$ to $p = 0.95$. In particular, at the middle of the band, the transmission goes from a dip to a peak. We also start seeing more sharp variations in the transmission curve beginning at $p = 0.95$. For very low amount of disorder, i.e., for $p = 0.99$, the fluctuations are more pronounced and the peak at the middle of the band is near the full transmission value of $T = 1$. In our previous paper [1] we have shown that in the case of no disorder, i.e., for $p = 1$, resonance occurs whenever the energy of the incident particle is close to a doubly degenerate eigenvalue of the square lattice without the leads. Thus, the pronounced variations that we see in Fig. 3 for low amounts of disorder are actually remnants of the resonances that exist at the ordered limit.

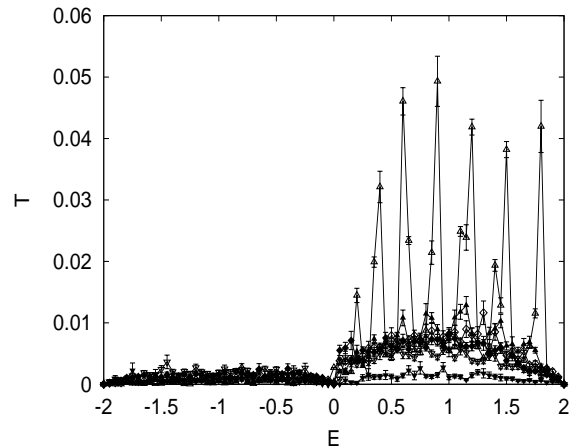


FIG. 4: Plot of the transmission coefficient T against the incident particle's energy E for disordered clusters on 20×20 lattices with the busbar connection between the leads and the cluster. The disorder concentrations are $p = 0.60$ (\blacktriangledown), $p = 0.70$ (\triangledown), $p = 0.80$ (\blacklozenge), $p = 0.90$ (\diamond), $p = 0.95$ (\blacktriangle), and $p = 0.99$ (\triangle). Notice that the transmission values are very small.

Quantum interference occurs within the disordered cluster and also at the connections between the leads and the cluster. With the busbar, the multiple connections between the cluster and the leads enhance the effect of quantum interference. An outcome of this is a significant decrease in the transmission of the particle. Shown in Fig. 4 is a plot of the transmission coefficient T as a function of the energy E of the incident particle for the busbar connection. Notice that the scale of the transmission only goes up to $T = 0.06$. For highly disordered clusters, just like in the point-to-point contacts case, the shape of the transmission curve resembles an inverted w with the dip at the middle of the band. As the amount of disorder is decreased we see a growing asymmetry between the positive and negative sides of E . For very low amount of disorder, i.e., for $p = 0.99$, there are pronounced variations in the transmission. These variations are remnants of the resonances at the ordered limit of a square lattice connected to leads through a busbar [1].

We now investigate how the transmission behaves as the disorder concentration is varied while the energy of the particle is held fixed. Shown in Fig. 5 is a plot of the transmission coefficient T as a function of the site occupation concentration p for the point-to-point contacts type of connection. The results are from the 20×20 lattices.

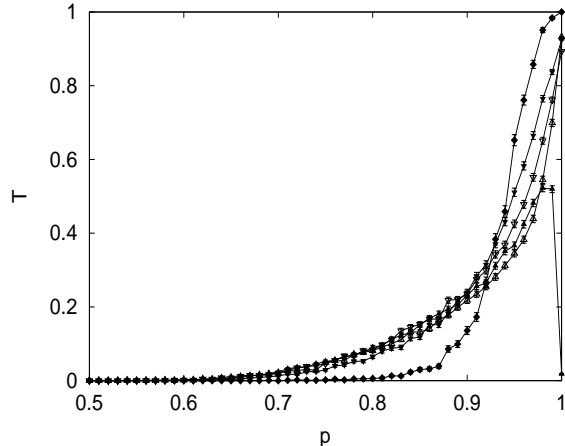


FIG. 5: Plot of the transmission coefficient T as a function of the cluster occupation concentration p for disordered clusters on 20×20 lattices with point-to-point contacts between the leads and the cluster. The incident particle has energies $E = 0.00$ (\blacklozenge), $E = 0.25$ (\triangle), $E = 0.50$ (\blacktriangle), $E = 0.75$ (∇), and $E = 1.00$ (\blacktriangledown).

The plot in Fig. 5 is for five selected values of the particle's energy E . These values are all positives since as we have seen in Fig. 3 there is symmetry between the positive and negative sides of E when the connection is point-to-point contacts. In Fig. 5 notice that the $E = 0$ case is special. For high disorder concentrations $p = 0.50$ until around $p = 0.90$ the curves for the other E values follow the same pattern. The system begins to be transmitting slightly above $p = 0.6$. Near $p = 0.90$ the transmission curves begin to spread out and at the region near $p = 1$ there are sharp variations as the system seems to change abruptly from one dominated by disorder to one that reflects the ordered limit wherein resonances occur. For the system with $E = 0$ it does not begin to transmit until around $p = 0.80$. The transmission curve then goes up with a slope that is steeper than the curves for the other E values. In Fig. 3 we have seen that at the middle of the band the transmission shifts from being a dip to a peak as p is varied. This shift in the behavior of the transmission is also manifest in the rapid rise of the transmission curve after $p = 0.90$ for the $E = 0$ case in Fig. 5.

For the busbar connection, we have seen in Fig. 4 that there is no symmetry between the positive and negative sides of the energy E . We therefore show in Figs. 6 and 7 plots of the transmission coefficient T as a function of the occupation concentration p as the energy E of the incident particle is fixed at either negative or positive values,

respectively. The system with the busbars is highly reflecting and we notice therefore in Figs. 6 and 7 that the transmission scales are only up to $T = 0.02$. Just like in the case for the point-to-point contacts, the $E = 0$ case appears to be special.

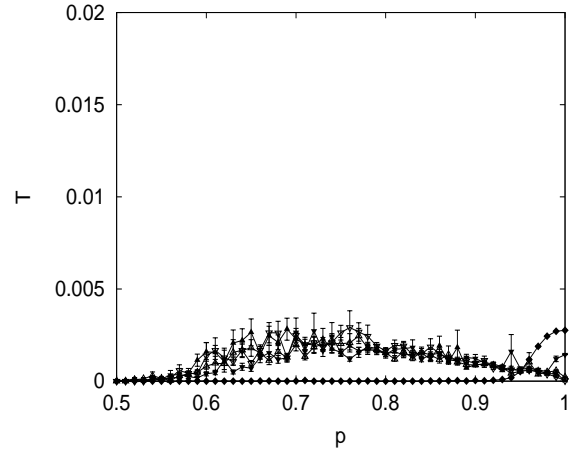


FIG. 6: Plot of the transmission coefficient T as a function of the cluster occupation concentration p for disordered clusters on 20×20 lattices with busbar connections between the leads and the cluster. The incident particle has energies $E = 0.00$ (\blacklozenge), $E = -0.25$ (\triangle), $E = -0.50$ (\blacktriangle), $E = -0.75$ (∇), and $E = -1.00$ (\blacktriangledown).

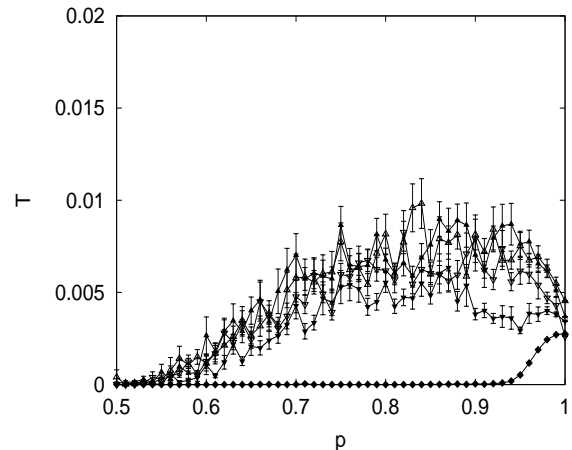


FIG. 7: Plot of the transmission coefficient T as a function of the cluster occupation concentration p for disordered clusters on 20×20 lattices with busbar connections between the leads and the cluster. The incident particle has energies $E = 0.00$ (\blacklozenge), $E = 0.25$ (\triangle), $E = 0.50$ (\blacktriangle), $E = 0.75$ (∇), and $E = 1.00$ (\blacktriangledown).

In Fig. 6 the selected energies are negative while in Fig. 7 the energies are positive. The data points for $E = 0$ are shown in both figures. Notice again that these points do not follow the same pattern as the data points for the other energies. In particular, the system is non-transmitting for high disorder concentrations until

near $p = 0.90$. For the other E values the system begins to transmit near $p = 0.60$. After about $p = 0.90$ the transmission curves spread out just like in the case for point-to-point contacts.

For both the busbar and point-to-point contacts we thus see signatures of a shift in transmission characteristics near $p_\alpha = 0.60$ and then again near $p_\beta = 0.90$. The system begins to transmit near p_α regardless of the type of connection chosen between the cluster and the leads, except at the middle of the band. Note that this value of p is close to the classical percolation threshold p_c of site percolation on a square lattice. In order for the particle to transmit through a disordered cluster that cluster must at least be spanning. In an infinite system, a spanning cluster only appears above the classical p_c . However, due to quantum interference and the finite sizes of the systems we study, the configuration will not necessarily begin transmitting exactly at p_c . The shift near p_β signals the onset of the dependence of the transmission coefficient T on the particle's energy E as the occupation concentration p is held fixed. Between p_α and p_β the transmission behaves in the same fashion, except, again, at the middle of the band. This can also be seen in Fig. 3 where for a high disorder concentration, say for $p = 0.80$, the transmission along the interval $0.25 < E < 1.75$ is relatively flat.

We further investigate the characteristics of the transmission as the size of the lattices are varied. Shown in Figs. 8 and 9 are plots of the transmission as a function of lattice sizes as the occupation concentration p and energy E of the incident particle are both held fixed. Also shown in the plots are the best-fitting exponential curves and best-fitting power-law curves.

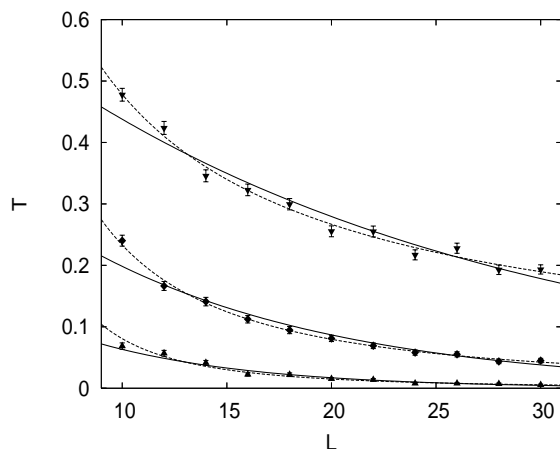


FIG. 8: Plot of the transmission coefficient T as a function of the lattice length L . The energy of the incident particle is $E = 1.00$. The cluster and the leads are connected through point-to-point contacts. The occupation concentrations are $p = 0.68$ (▲), $p = 0.80$ (◆), and $p = 0.92$ (▼). The solid lines are the best-fitting exponential curves while the dashed lines are the best-fitting power-law curves.

In Fig. 8 we have chosen the energy of the particle

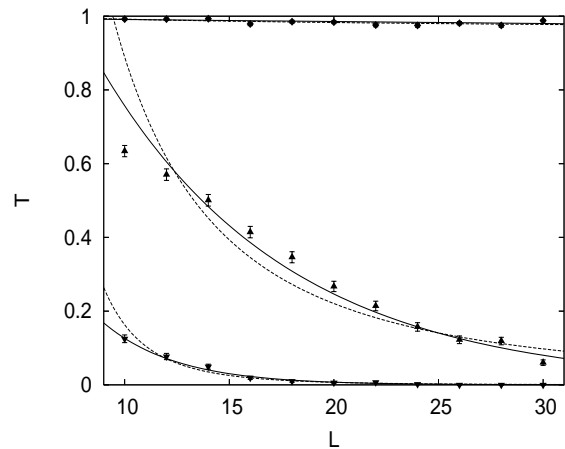


FIG. 9: The transmission coefficient T as a function of the lattice length L with an incident particle of energy $E = 0.00$. The occupation concentrations are $p = 0.80$ (▼), $p = 0.92$ (▲) and $p = 0.99$ (◆). The solid lines are the best-fitting exponential curves while the dashed lines are the best-fitting power-law curves. For $p = 0.80$ the three points after $L = 25$ have transmissions that are practically zeroes and are therefore not included when the best-fit lines are determined.

as $E = 1$. We have also chosen the connection between the cluster and the leads to be point-to-point contacts because this connection type is generally more transmitting than the busbar. The cluster occupation concentrations are chosen as $p = 0.68$ and $p = 0.92$ since these values are near p_α and p_β . The power-law fits are in the form $T = T_p L^{-\sigma_p}$ while the exponential fits are in the form $T = T_e e^{-\sigma_e L}$, for lattices of size $L \times L$. Shown in Table I are the fitting values corresponding to the best-fit curves in Fig. 8. The R_p and R_e are the correlation coefficients for the power-law and exponential fits performed as linear regressions of $\log(T)$ against $\log(L)$ and $\log(T)$ against L , respectively.

p	T_p	σ_p	$ R_p ^2$	T_e	σ_e	$ R_e ^2$
0.68	21.67	2.429	0.97911	0.234	0.131	0.97001
0.80	8.249	1.549	0.99455	0.453	0.083	0.96994
0.92	3.323	0.842	0.98331	0.686	0.045	0.95684

TABLE I: Table of fitting values for the $E = 1$ case. The T_p , σ_p , and R_p belong to the power-law fits while the T_e , σ_e , and R_e are for the exponential fits.

From the values of the correlation coefficients, R_p and R_e , it is not possible to clearly distinguish between the goodness of the power-law and exponential fits as they are very close to each other. However, since these correlation coefficients are reasonably close to 1, we can be fairly confident that T extrapolates to zero as $L \rightarrow \infty$ or in the thermodynamic limit. Thus, in either case, our result for $E = 1$ is consistent with one-parameter scaling theory [2] which predicts that wavefunctions in two

dimensions are localized.

Shown in Fig. 9 is a plot of T as a function of L when the particle's energy is $E = 0$. The corresponding best-fit estimates are shown in Table II. While the power-law and exponential curves are both reasonably good for $p = 0.80$, the power-law fit is not nearly as reasonable for $p = 0.92$. For very low amount of disorder at $p = 0.99$ neither the power-law nor the exponential fit is reasonable. From the values of σ_{0p} and σ_{0e} we get a hint that the T can be independent of L at very low disorder concentrations, at least at certain values of E . For such a situation, it is possible for the transmission to be non-zero even for asymptotically large clusters.

p	T_{0p}	σ_{0p}	$ R_{0p} ^2$	T_{0e}	σ_{0e}	$ R_{0e} ^2$
0.80	6122	4.575	0.95824	2.185	0.285	0.96564
0.92	91.2	2.011	0.90850	2.325	0.112	0.96878
0.99	1.02	0.013	0.44440	0.997	0.001	0.37233

TABLE II: Table of fitting values for the $E = 0$ case. The T_{0p} , σ_{0p} , and R_{0p} belong to the power-law fits while the T_{0e} , σ_{0e} , and R_{0e} are for the exponential fits. Note that neither fits are good for the $p = 0.99$ case.

IV. SUMMARY AND CONCLUSIONS

The input and output leads can be attached to the disordered cluster in several different ways. In this paper we have chosen the connections to be point-to-point contacts and the busbar. The behavior of the transmission T is independent of the type of connection chosen for highly disordered clusters. This is because the number of actual multiple connections in the busbar case is not large enough to differentiate its effects from that of the point-to-point contacts type of connection. As the site dilution is diminished, i.e., as p increases, the effects of the choice of connection begin to be apparent. For low disorder, i.e., high p , point-to-point contacts, in general, are more transmitting than the busbar. The resonances at the ordered limit, however, influence the transmission at low disorder. We thus see transmission variations near $p = 1$ as we vary the energy of the particle.

We have shown how the transmission coefficient T behaves as a function of the energy E of the particle, the occupation concentration p of the disordered cluster, and the size $L \times L$ of the underlying lattice. For 20×20 lattices we find the system to begin transmitting around $p_\alpha = 0.60$, which is near the classical percolation threshold p_c . The transmission is independent of the incident

particle's energy along the interval $0.25 \leq E \leq 1.75$ when p is increased until around $p_\beta = 0.90$. After p_β the transmission begins to be dependent on the energy of the particle. The exception to this pattern is when the particle has energy at the middle of the band. For such a particle the system does not begin to be transmitting until after around $p = 0.80$ in point-to-point contacts and $p = 0.90$ in the busbar.

Extrapolating the behavior of the transmission to very large lattices, we find $T \rightarrow 0$ in moderate to high disorder. The system is localized in these situations. The behavior of the transmission at very low disorder, however, is not as straightforward. The resonances at the ordered limit influences the behavior of the transmission at very low disorder. So even with asymptotically large clusters, as long as the disorder is low, it may be possible to get non-zero transmission when the energy of the incident particle is close to the resonances at the ordered limit. When the energy of the incident particle is at the middle of the band, the system can also be transmitting at very low disorder even for asymptotically large clusters. This case is consistent with the work of Inui *et al.* [24] where they cite the bipartite symmetry of the underlying square lattice as the reason for finding wavefunctions that are not exponentially localized at the middle of the band.

Quantum percolation in two dimensions in asymptotically large clusters therefore generally lead to non-conducting systems. Our results suggest that the only possible exceptions occur at very low disorder when the incident particle's energy is either at the middle of the band or near the resonance value in the ordered limit. Quantum percolation is a single-particle quantum model of hopping transport where the effects from mechanisms such as tunneling, long-ranged hopping, or inter-particle interactions are not taken into account. Taking these other effects into account may enhance the transmission of the particle in such a way that the configuration may be transmitting even for asymptotically large and highly disordered clusters.

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